

Chapter 3

Modelling the climate system



Description of the different types of climate models and of their components

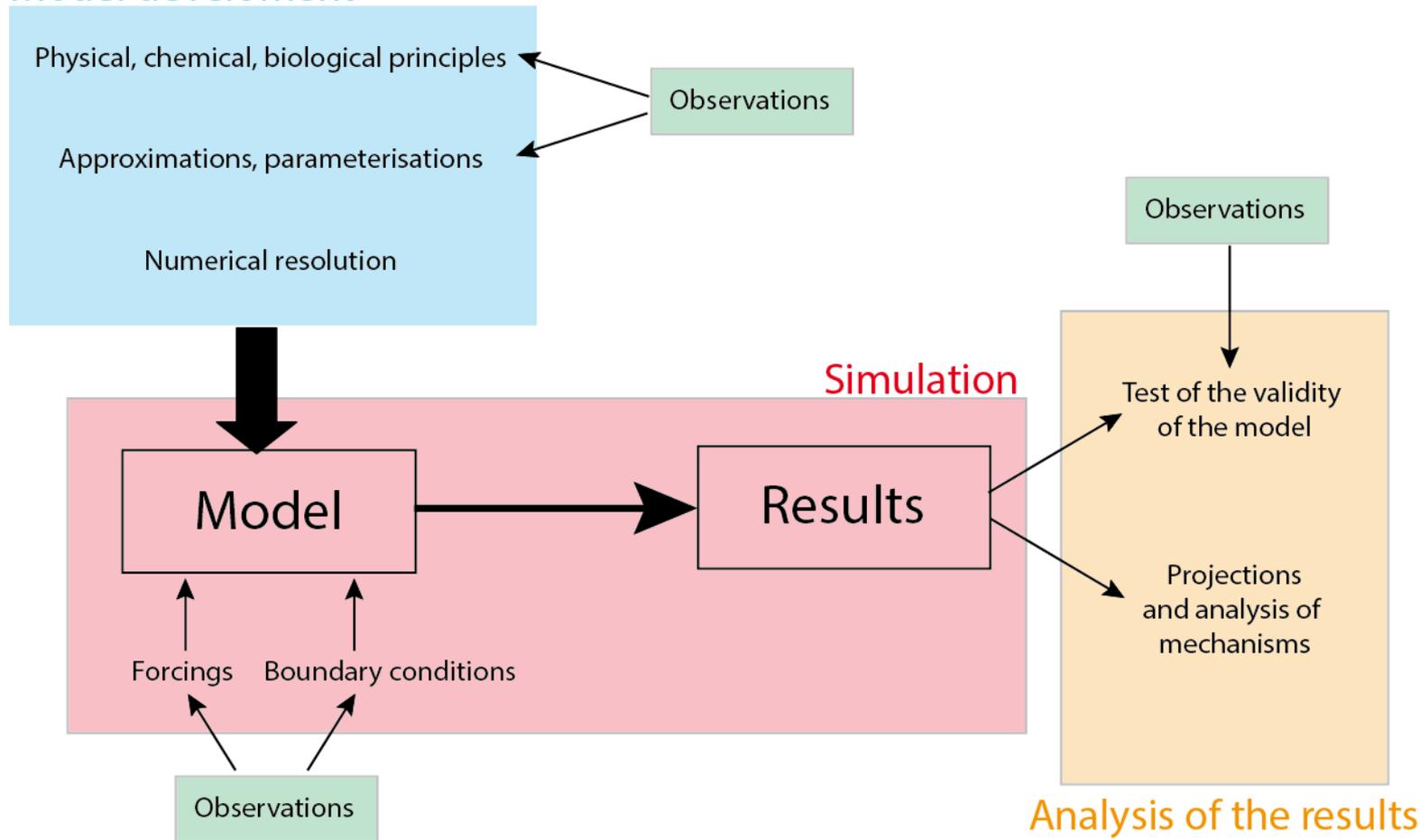
Test of the performance of the models

Interpretation of model results in conjunction with observations

Climate models

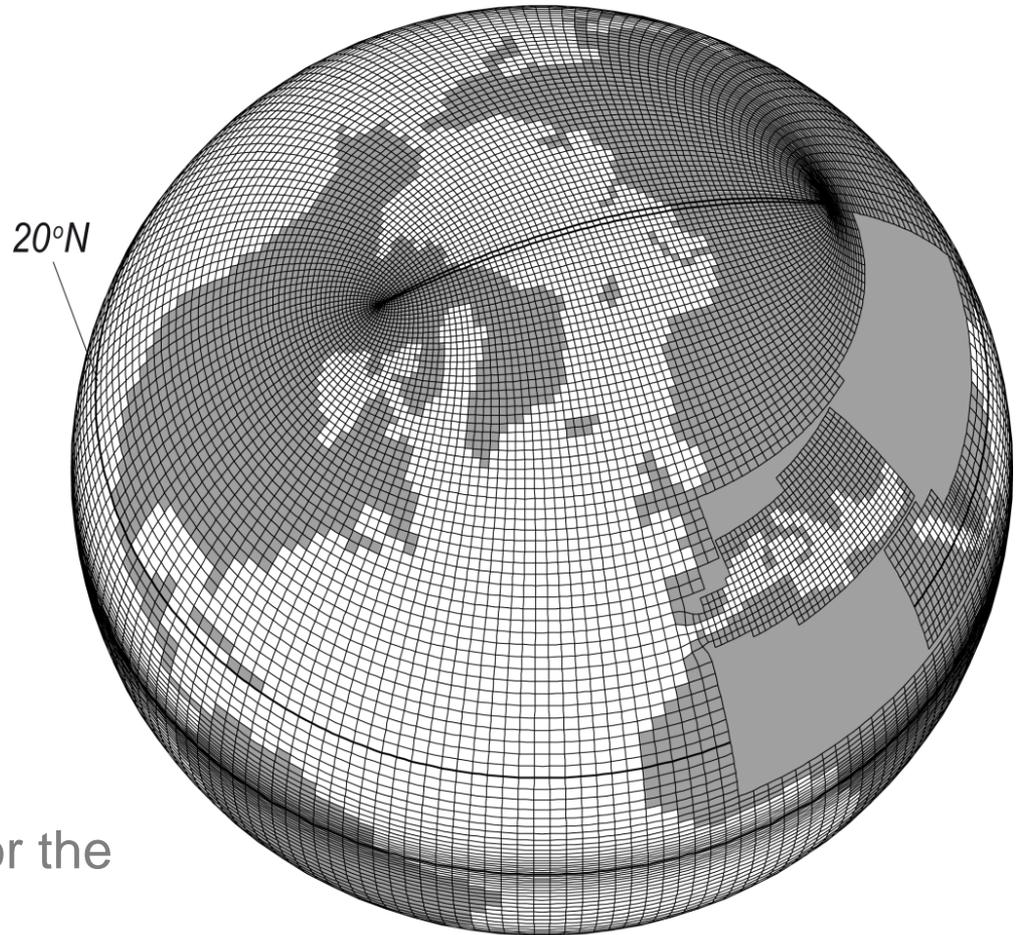
A **climate model** is based on a **mathematical** representation of the climate system derived from **physical, biological and chemical principles** and on the way the resulting equations are **solved**.

Model development



Climate models

The **equations** included in climate models are so complex that they must be solved **numerically**, generally on a **numerical grid**.



Example of a numerical grid for the ocean model NEMO.

Parameterisations account for the large-scale influence of processes not included explicitly.

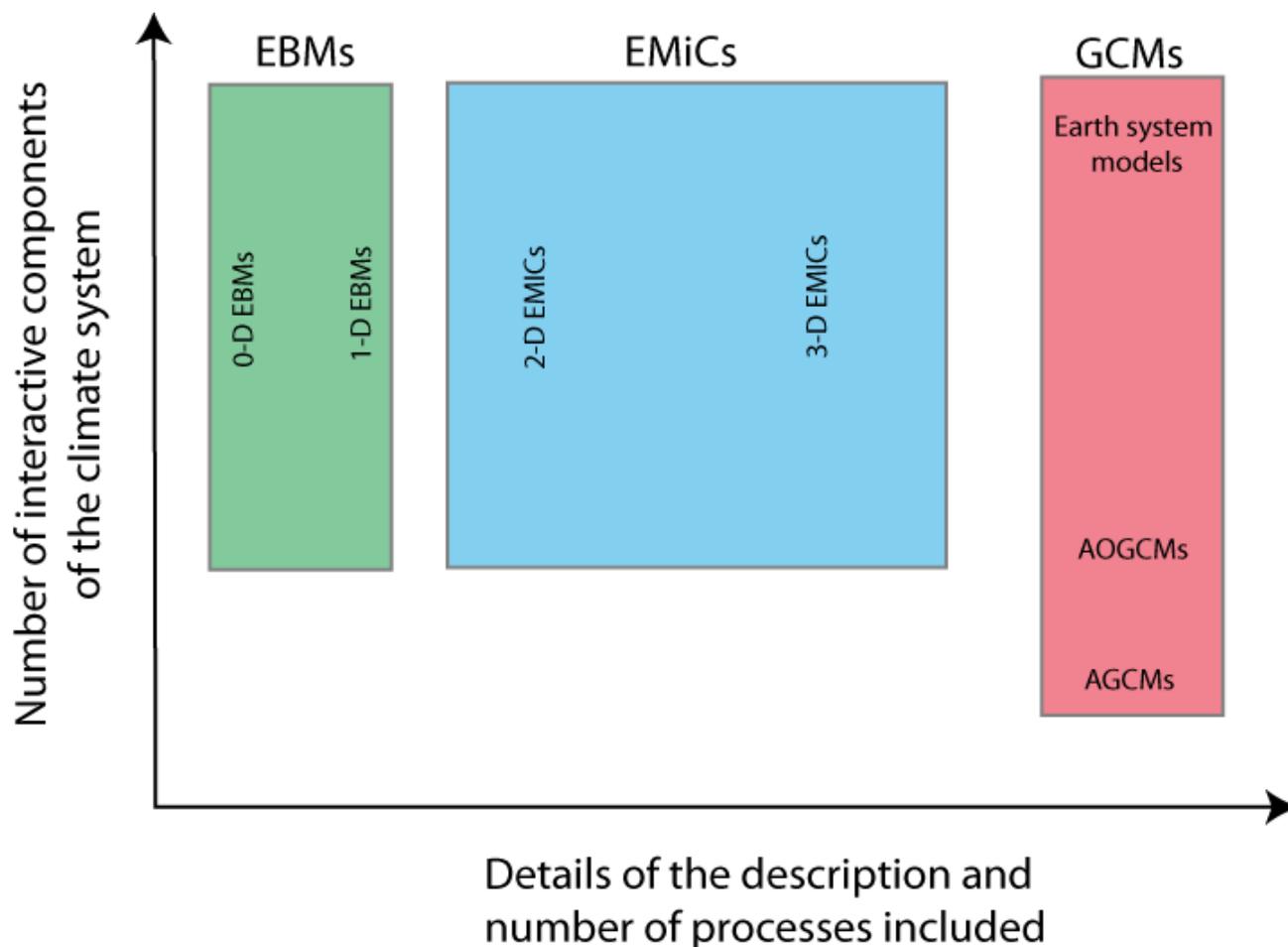
Climate models require some **inputs** from observations or other model studies.

They are often separated in

- **boundary conditions** which are generally fixed
- **external forcings** which drives climate changes.

Climate models

Each model type has its own specific purpose. The different types are complementary.



Energy balance models

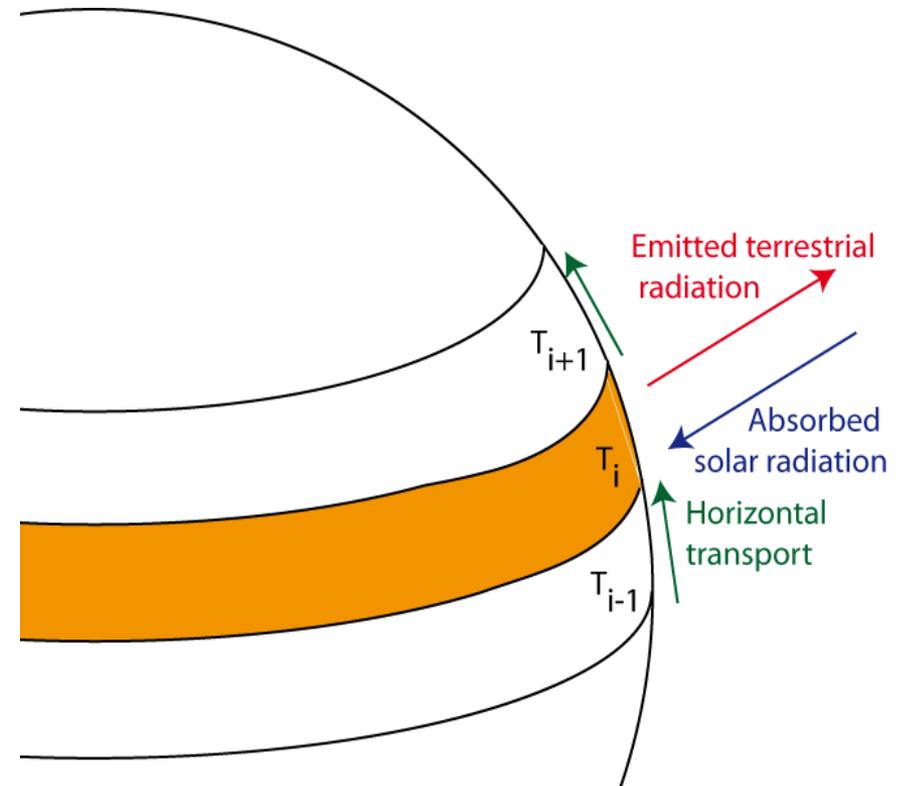
EBMs are based on a simple energy balance.

Changes in heat storage = absorbed solar radiation
- emitted terrestrial radiation (+transport)

$$C_m \frac{\partial T_i}{\partial t} = \left((1 - \alpha_i) \frac{S_i}{4} - A \uparrow \right) + \Delta transp$$

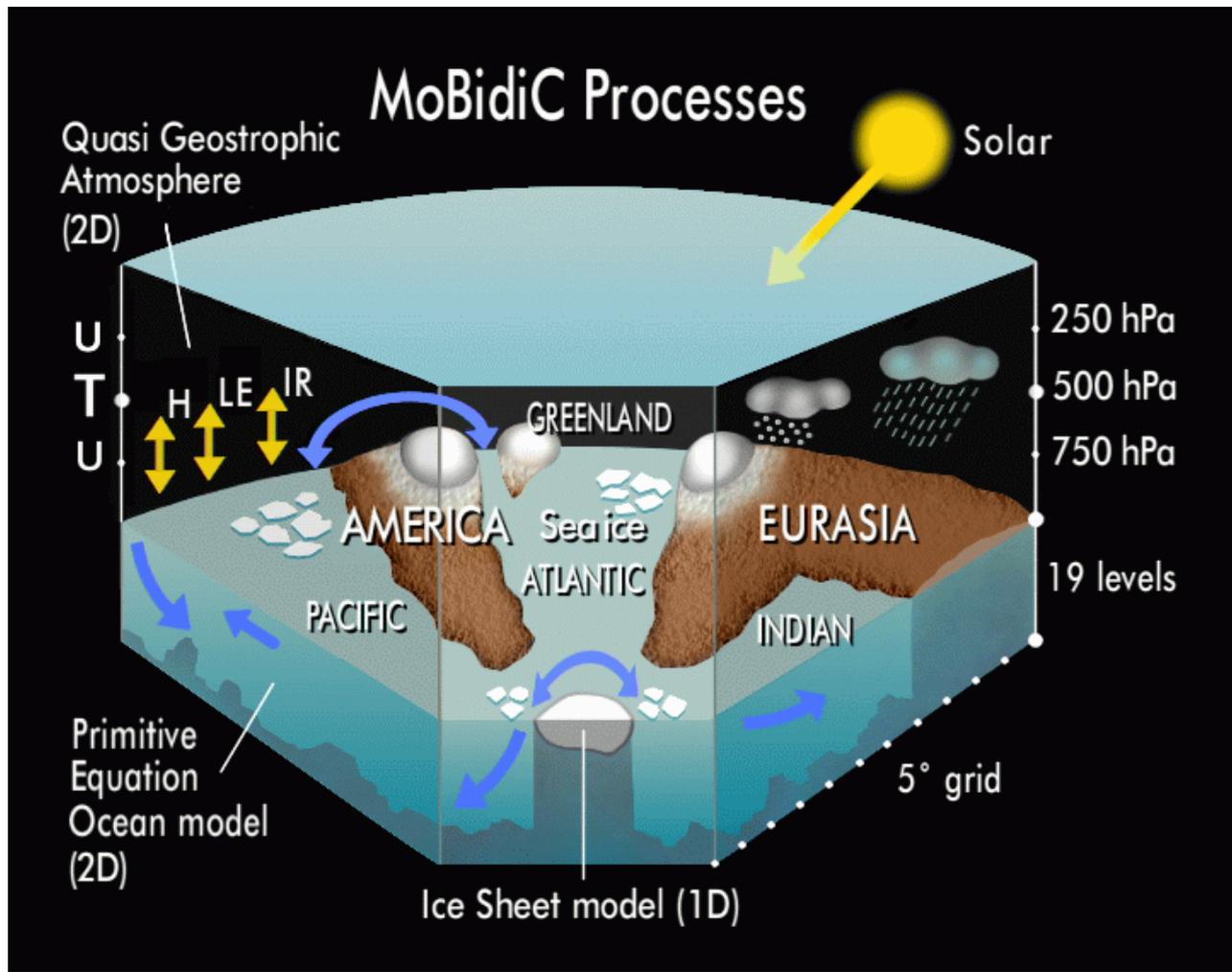
$$A \uparrow = \varepsilon \sigma T_s^4 \tau_a$$

τ_a represents the infrared transmissivity of the atmosphere (including the greenhouse gas effect)



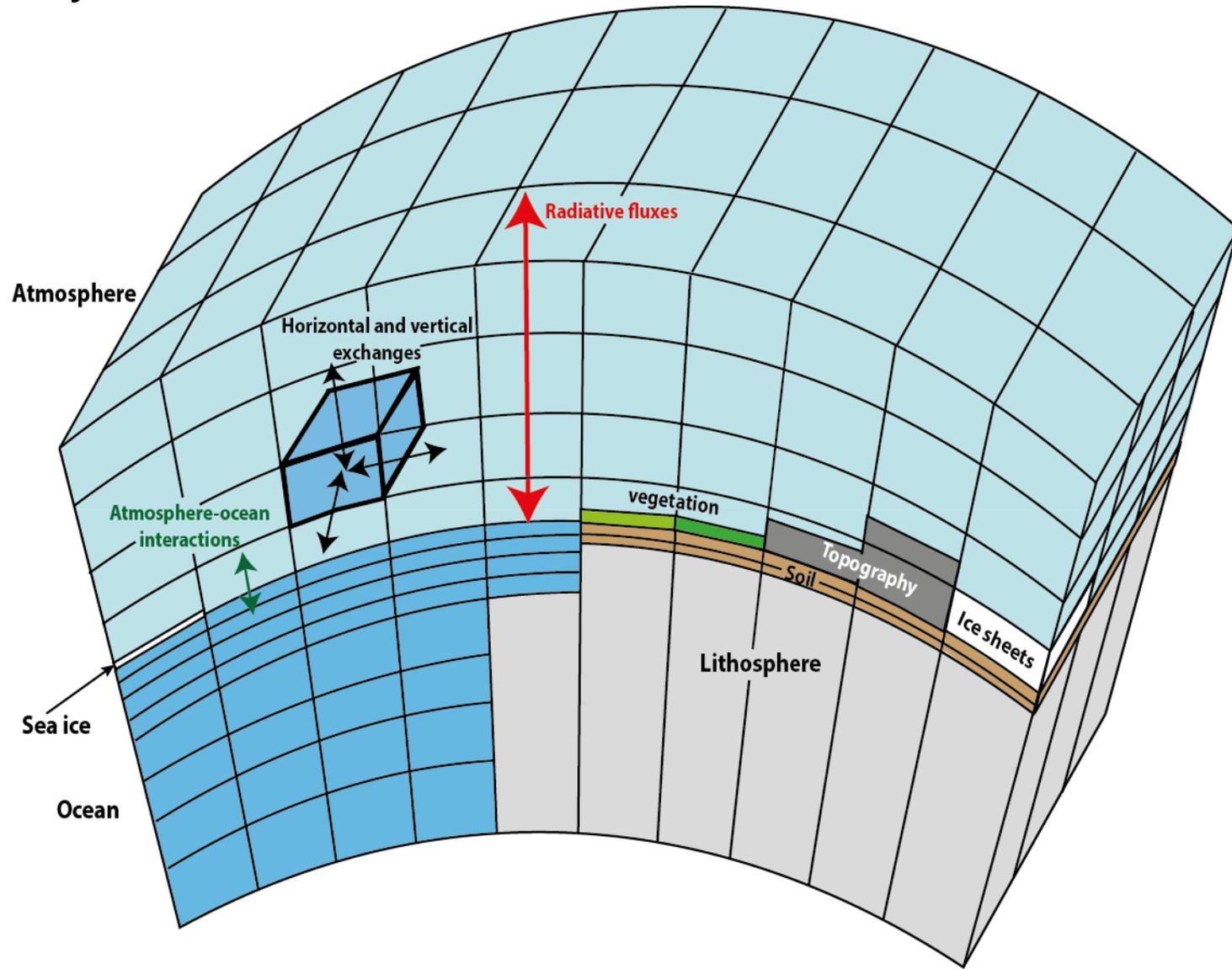
Earth Models of Intermediate Complexity

EMICs involve some simplifications, but they always include a representation of the Earth's geography.



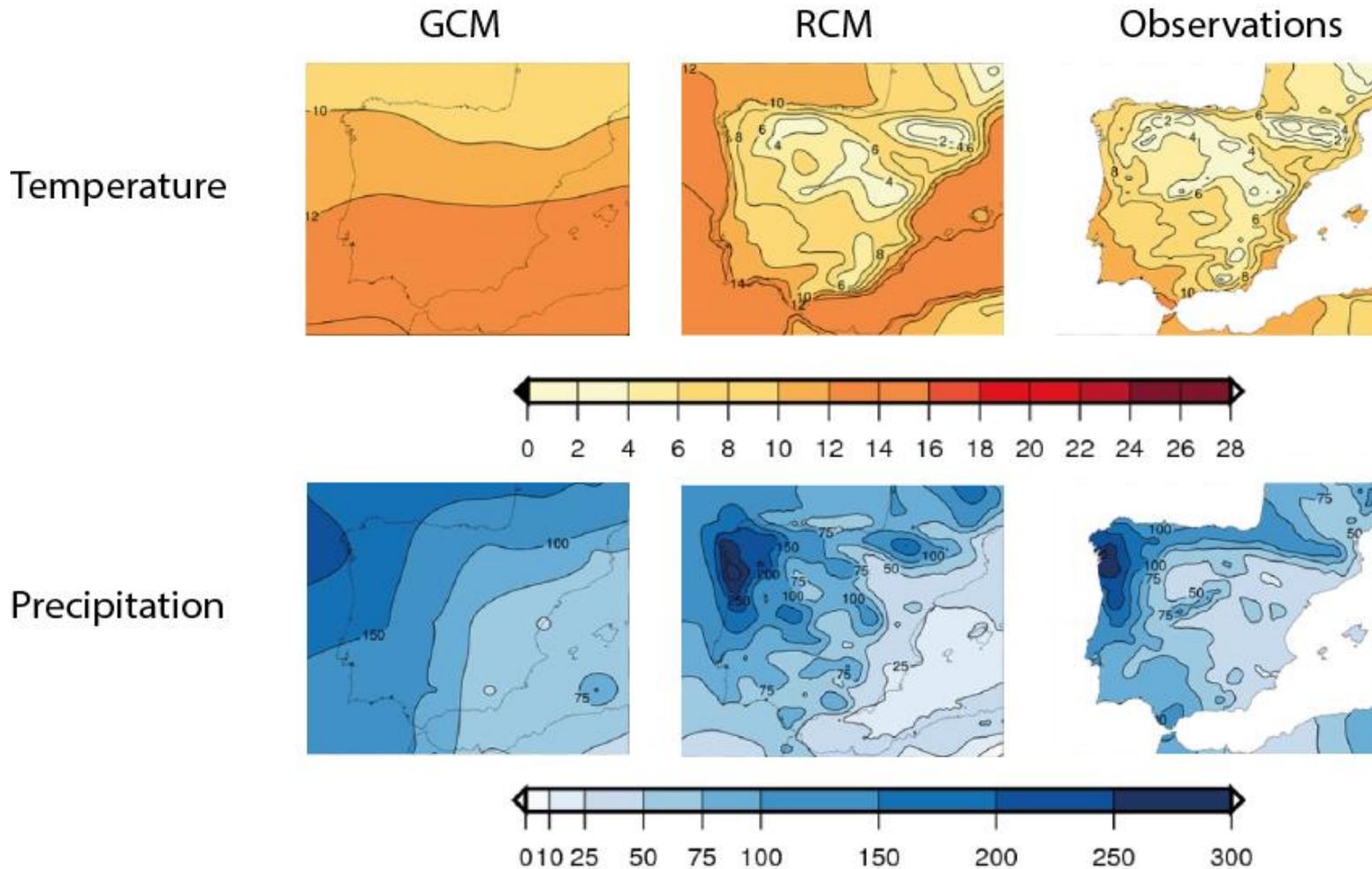
General circulation models

GCMs provide the most precise and complex description of the climate system.



Regional climate models

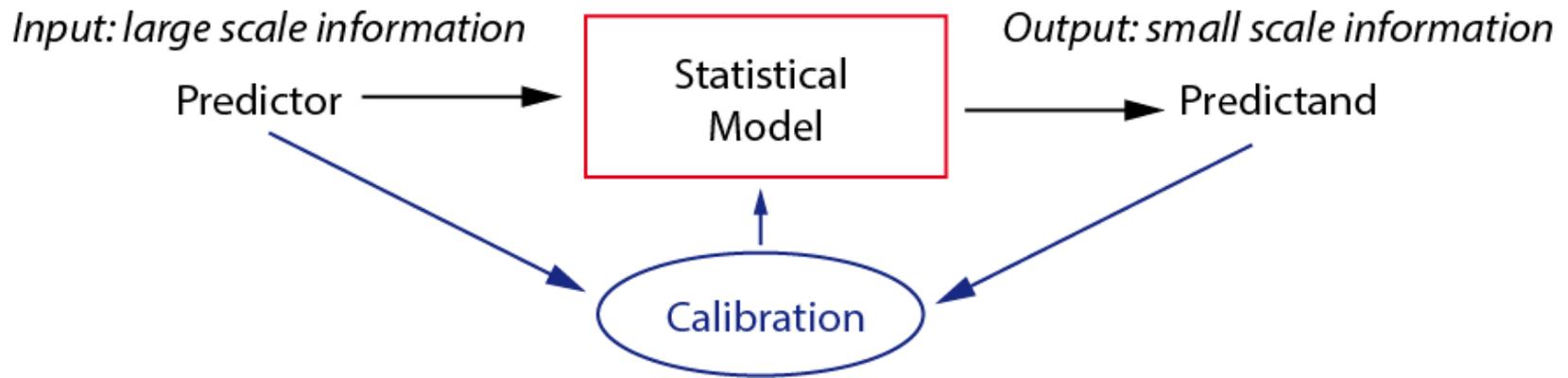
RCMs allow a more detailed investigation over a smaller domain.



Winter temperature ($^{\circ}\text{C}$) and precipitation (mm month^{-1}) in a coarse resolution GCM with grid spacing of the order of 400 km, a RCM with a resolution of 30 km and in observations. Figure from Gómez-Navarro et al. (2011).

Statistical downscaling

Local/regional information is deduced from the results of a large scale climate model using a statistical approach.



The simplest formulation is a linear relationship between the predictand (T_{loc}) and the predictor (T_{GCM}).

$$T_{loc} = \alpha_{sd} T_{GCM} + \beta_{sd}$$

α_{sd} and β_{sd} are two coefficients.

Components of a climate model: atmosphere

The basic equations for the atmosphere are a set of **seven equations** with seven unknowns.

The unknowns are the three components of the **velocity** (components u , v , w), the **pressure** p , the **temperature** T , the specific **humidity** q and the **density** ρ .

Components of a climate model: atmosphere

(1-3) Newton's second law

$$\frac{d\vec{v}}{dt} = -\frac{1}{\rho} \vec{\nabla} p + \vec{g} + \vec{F}_{fric} - 2\vec{\Omega} \times \vec{v}$$

Acceleration

Force due to pressure gradient

Gravitational force

Friction force

Coriolis force

In this equation, d/dt is the total derivative, including a transport term

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}$$

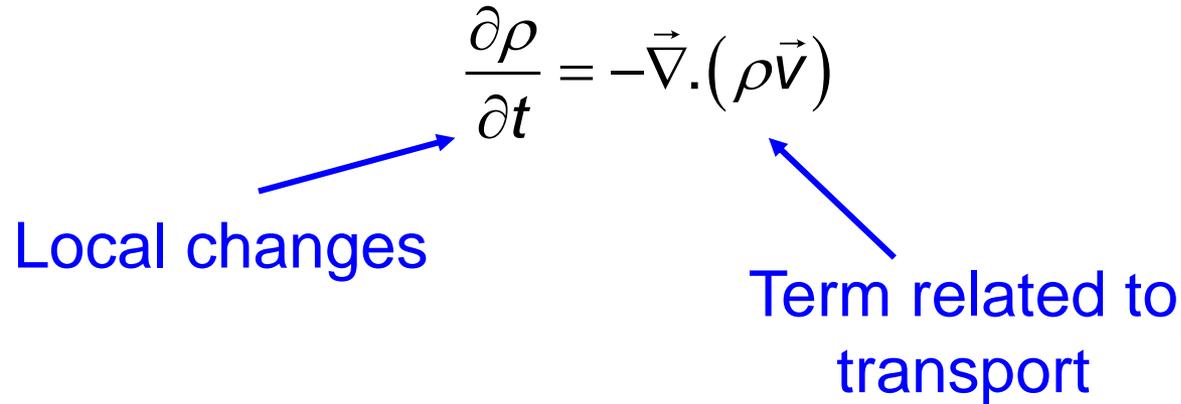
Components of a climate model: atmosphere

(4) The continuity equation or the conservation of mass

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot (\rho \vec{V})$$

Local changes

Term related to transport

The diagram shows the continuity equation $\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot (\rho \vec{V})$. A blue arrow points from the text "Local changes" to the partial derivative term $\frac{\partial \rho}{\partial t}$. Another blue arrow points from the text "Term related to transport" to the divergence term $-\vec{\nabla} \cdot (\rho \vec{V})$.

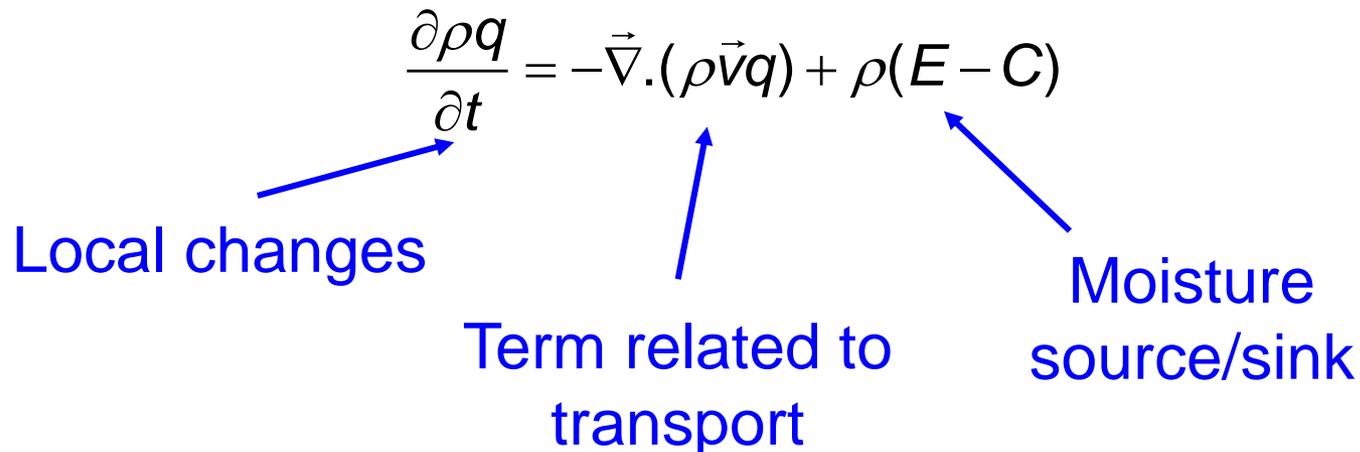
(5) The conservation of water vapour mass

$$\frac{\partial \rho q}{\partial t} = -\vec{\nabla} \cdot (\rho \vec{v} q) + \rho(E - C)$$

Local changes

Term related to transport

Moisture source/sink

The diagram shows the conservation of water vapour mass equation $\frac{\partial \rho q}{\partial t} = -\vec{\nabla} \cdot (\rho \vec{v} q) + \rho(E - C)$. A blue arrow points from the text "Local changes" to the partial derivative term $\frac{\partial \rho q}{\partial t}$. A second blue arrow points from the text "Term related to transport" to the divergence term $-\vec{\nabla} \cdot (\rho \vec{v} q)$. A third blue arrow points from the text "Moisture source/sink" to the source/sink term $\rho(E - C)$.

Components of a climate model: atmosphere

(6) The first law of thermodynamics (the conservation of energy)

$$C_p \frac{dT}{dt} = Q + \frac{1}{\rho} \frac{dp}{dt}$$

Changes in internal energy

Heat input

Work

The diagram shows the equation $C_p \frac{dT}{dt} = Q + \frac{1}{\rho} \frac{dp}{dt}$. Three blue arrows point from text labels to terms in the equation: one from 'Changes in internal energy' to $C_p \frac{dT}{dt}$, one from 'Heat input' to Q , and one from 'Work' to $\frac{1}{\rho} \frac{dp}{dt}$.

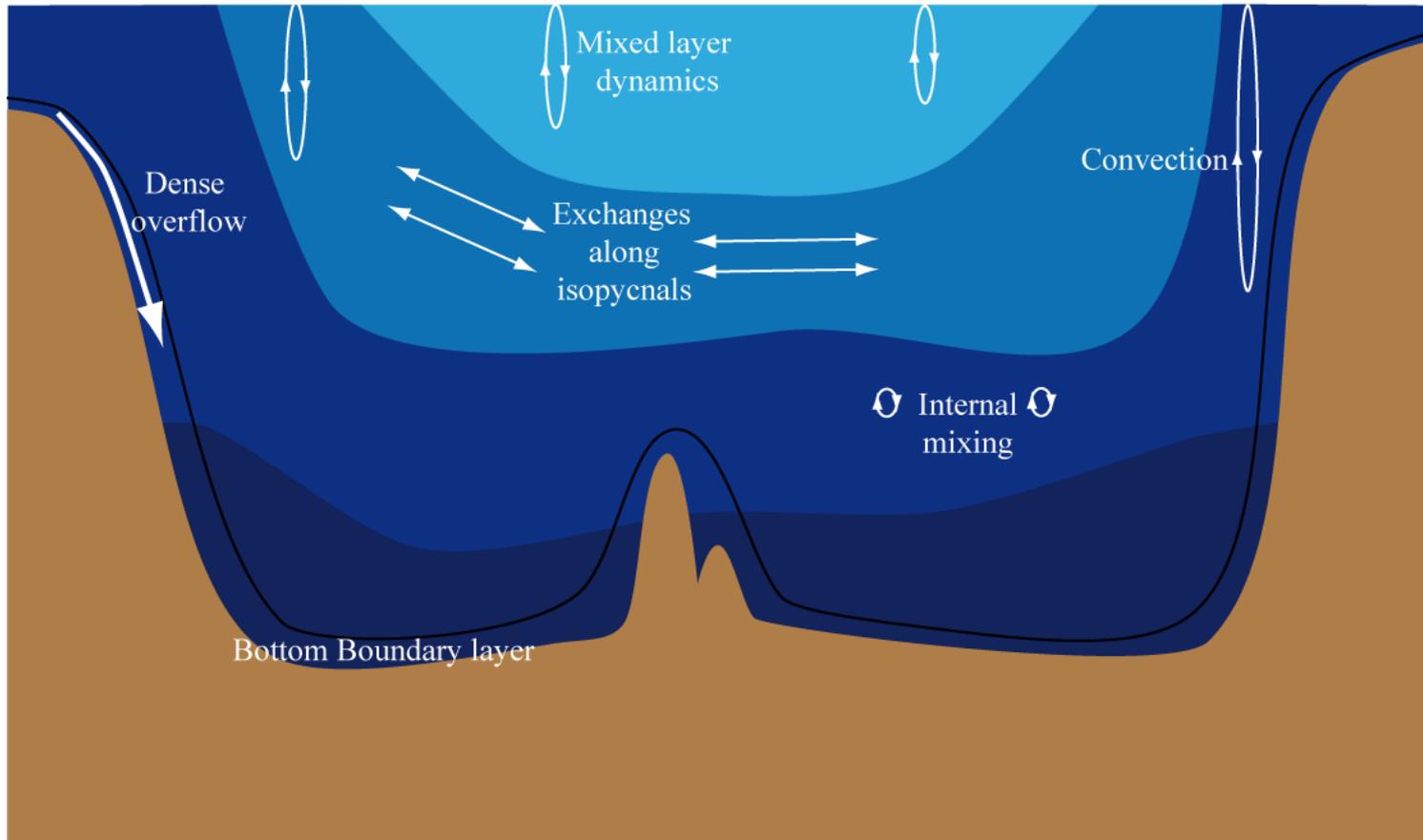
(7) The equation of state

$$p = \rho R_g T$$

+ model “physics”: parameterisation of subgrid-scale processes, radiative fluxes, turbulent fluxes, etc.

Components of a climate model: ocean

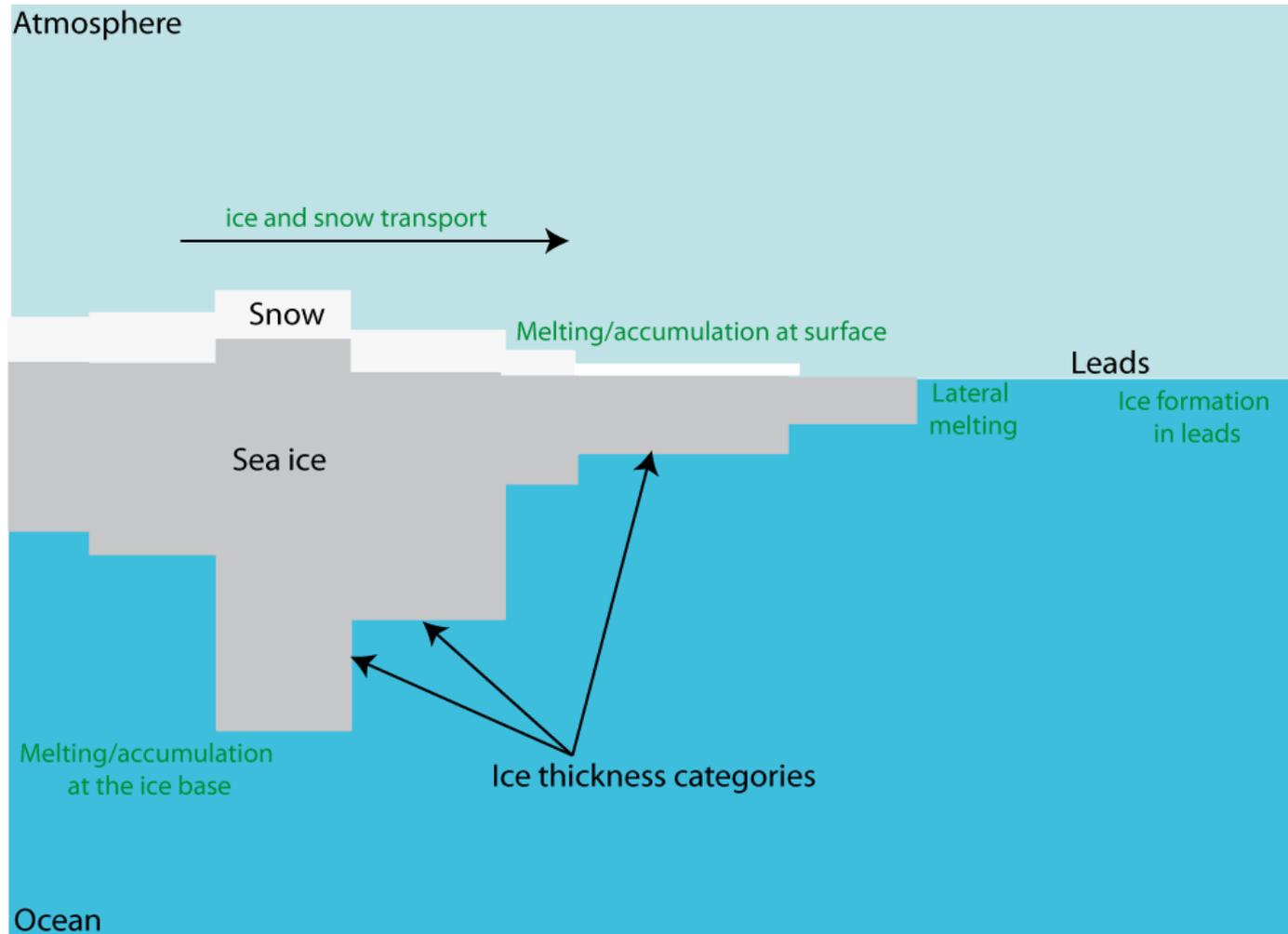
The equations for the ocean are similar to the ones for the atmosphere. The unknowns are the velocity, the density, the temperature and the salinity.



Some small-scale processes that have to be parameterised in global ocean models.

Components of a climate model: sea ice

The physical processes governing the development of sea ice can be conceptually **divided** into the **thermodynamic** growth or decay of the ice and the large-scale **dynamics** of sea ice.



Components of a climate model: sea ice

Thermodynamics

The temperature inside snow and ice (T_c) is computed from a one-dimensional equation:

$$\rho_c c_c \frac{\partial T_c}{\partial t} = k_c \frac{\partial^2 T_c}{\partial z^2}$$

Changes in internal energy \rightarrow $\rho_c c_c \frac{\partial T_c}{\partial t}$ \leftarrow Heat input due to diffusion $\frac{\partial^2 T_c}{\partial z^2}$

where ρ_c , c_c , and k_c are the density, specific heat and thermal conductivity

The surface and basal melting/accretion is deduced from the energy balance at the interfaces.

Components of a climate model: sea ice

Dynamics

Sea ice is modelled as a two-dimensional continuum.

$$m \frac{d\vec{u}_i}{dt} = \vec{\tau}_{ai} + \vec{\tau}_{wi} - m f \vec{e}_z \times \vec{u}_i - mg \vec{\nabla} \eta + \vec{F}_{\text{int}}$$

acceleration

Air and water drag

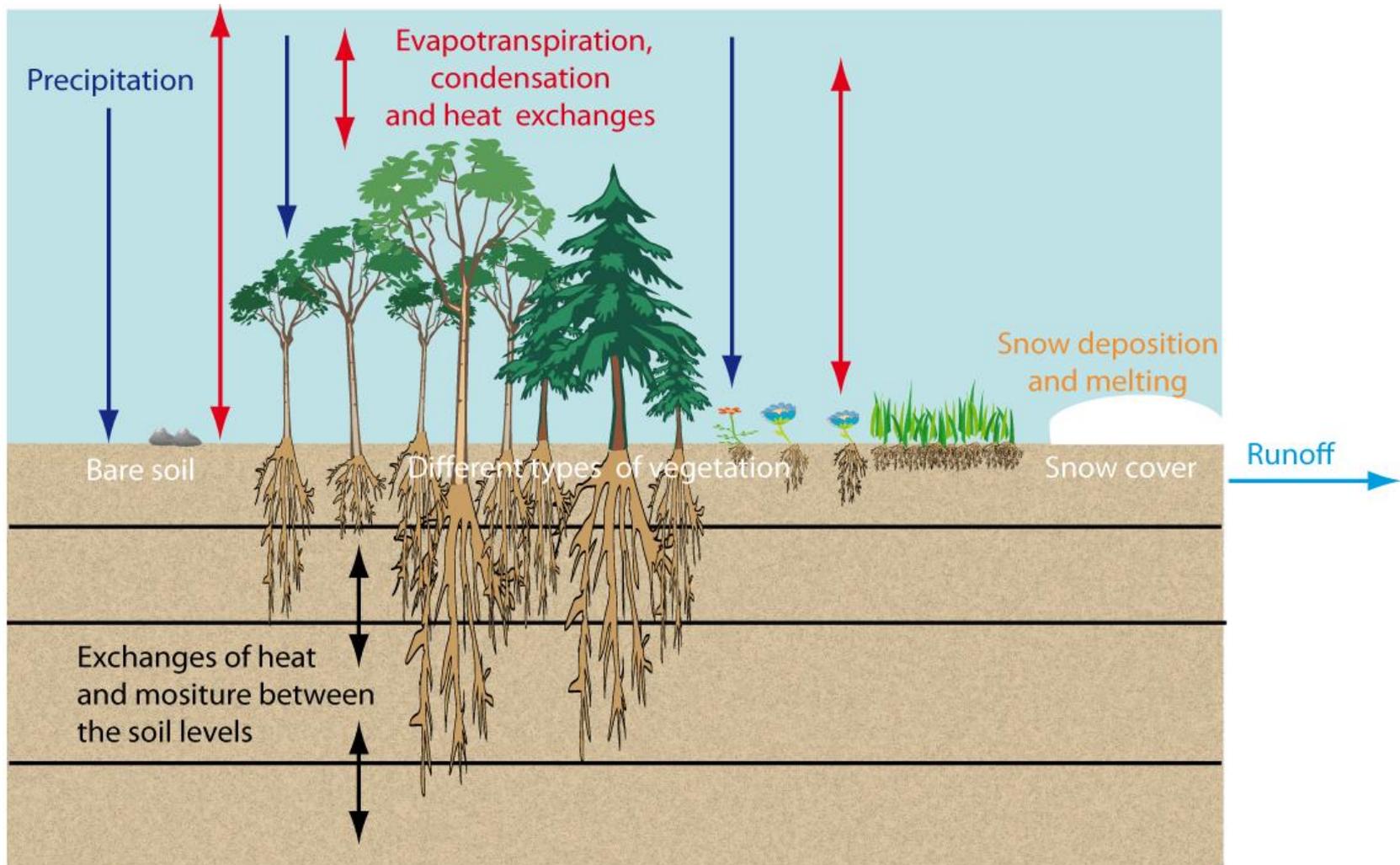
Coriolis force

Force due to the oceanic tilt

Internal forces

where m is the mass of snow and ice per unit area, \vec{u}_i is the ice velocity and f , \vec{e}_z , g and η are respectively the Coriolis parameter, a unit vector pointing upward, the gravitational acceleration g and the sea-surface elevation.

Components of a climate model: land surface



The main processes that have to be taken into account in a land surface model.

Components of a climate model: land surface

Computation of the surface temperature: energy balance of a thin surface layer of thickness h_{su} .

$$\rho c_p h_{su} \frac{\partial T_s}{\partial t} = (1 - \alpha) F_{sol} + F_{IR\downarrow} + F_{IR\uparrow} + F_{SE} + F_{LE} + F_{cond}$$

Changes in internal energy

Net solar flux

Downward and upward IR fluxes

Sensible heat flux

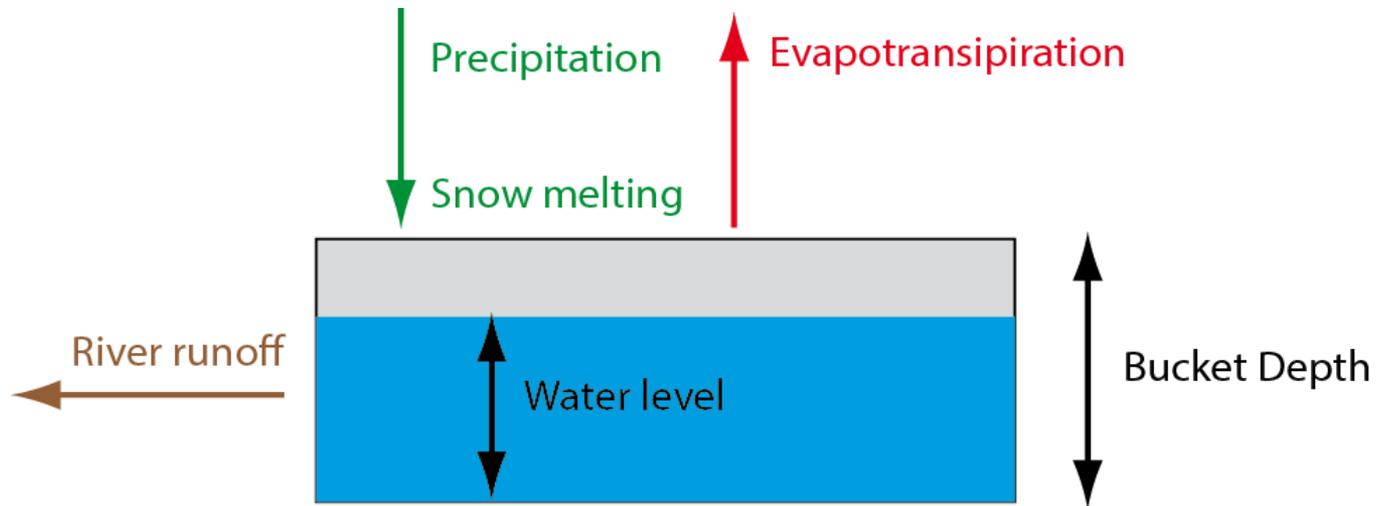
Latent heat flux

Heat conduction

+ snow melting/accumulation

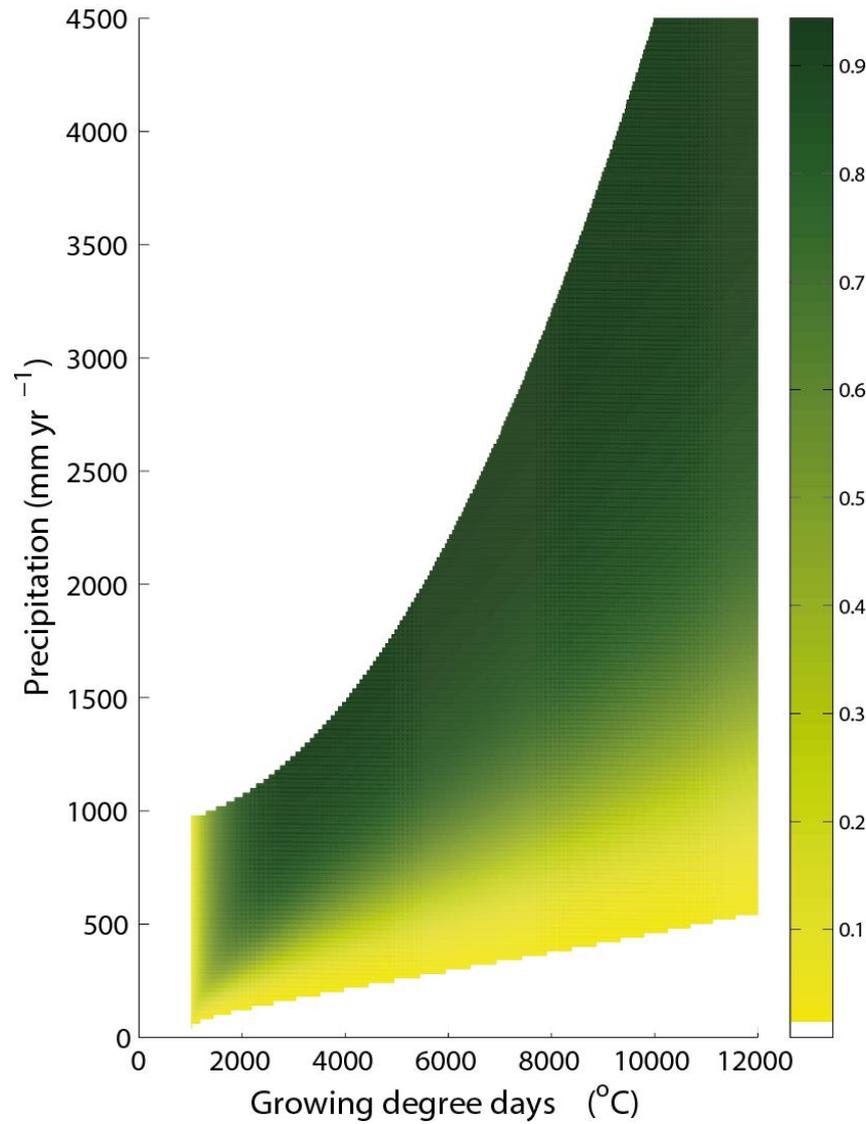
Components of a climate model: land surface

Land bucket model



Components of a climate model: land surface

Dynamic global vegetation models (DGVMs) compute the dynamics of the vegetation cover in response to climate changes



The equilibrium fraction of trees in a model that includes two plant functional types and whose community composition is only influenced by precipitation and the growing degree days (GDD)

Components of a model: marine biogeochemistry

Models of biogeochemical cycles in the oceans are based on a set of equations formulated as:

$$\frac{dTrac}{dt} = F_{diff} + Sources - Sinks$$

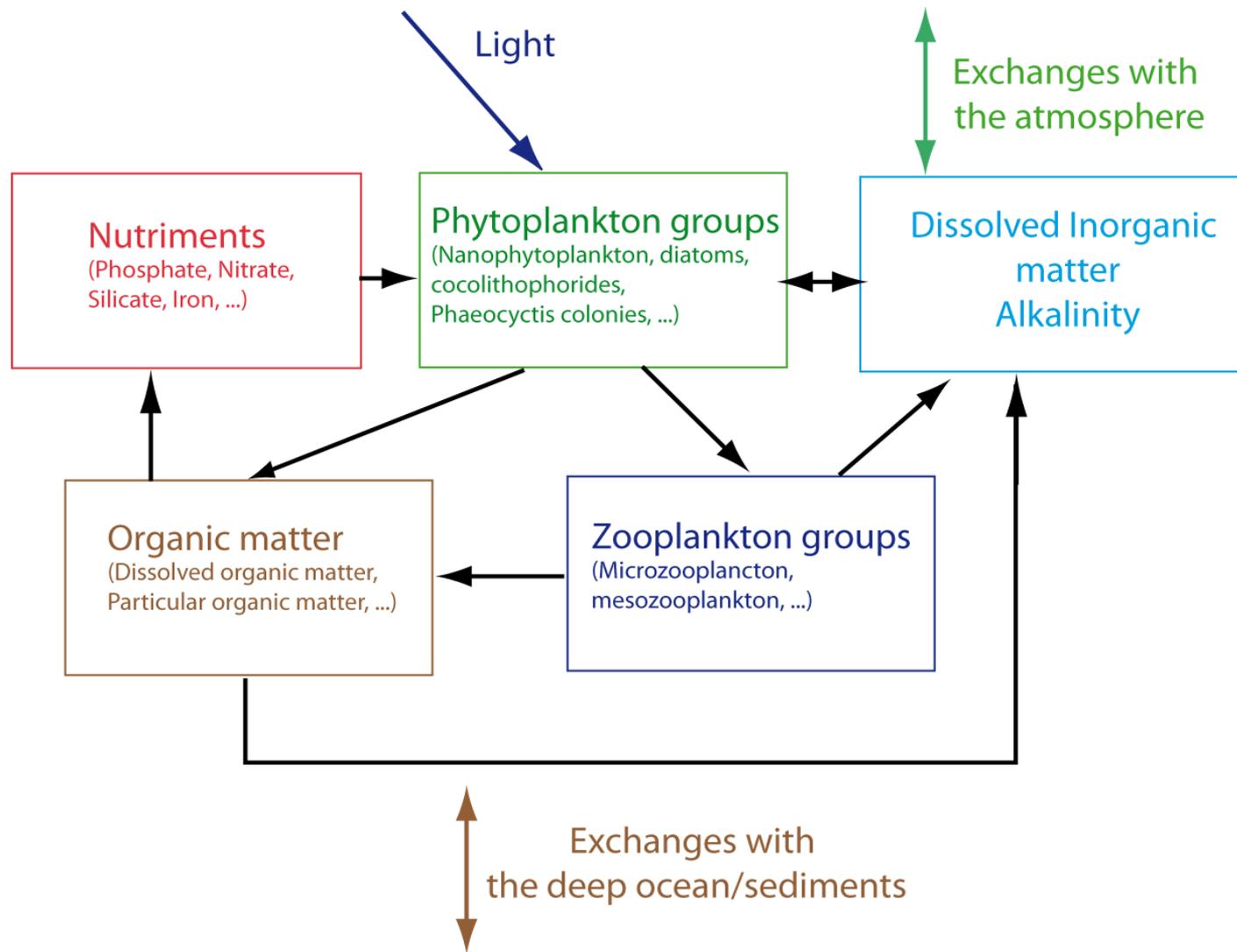
Local changes and transport by the flow

Diffusion

Biogeochemical processes

where *Trac* is a biogeochemical variable. Those variables are often called tracers because they are transported and diffused by the oceanic flow.

Components of a model: marine biogeochemistry



Some of the variables of a biogeochemical model.

Components of a climate model: ice sheets

Ice-sheet models can also be decomposed into a dynamic core and a thermodynamic part.

The conservation of ice volume, which relates both parts, can be written as

$$\frac{\partial H}{\partial t} = -\vec{\nabla} \cdot (\vec{u}_m H) + M_b$$

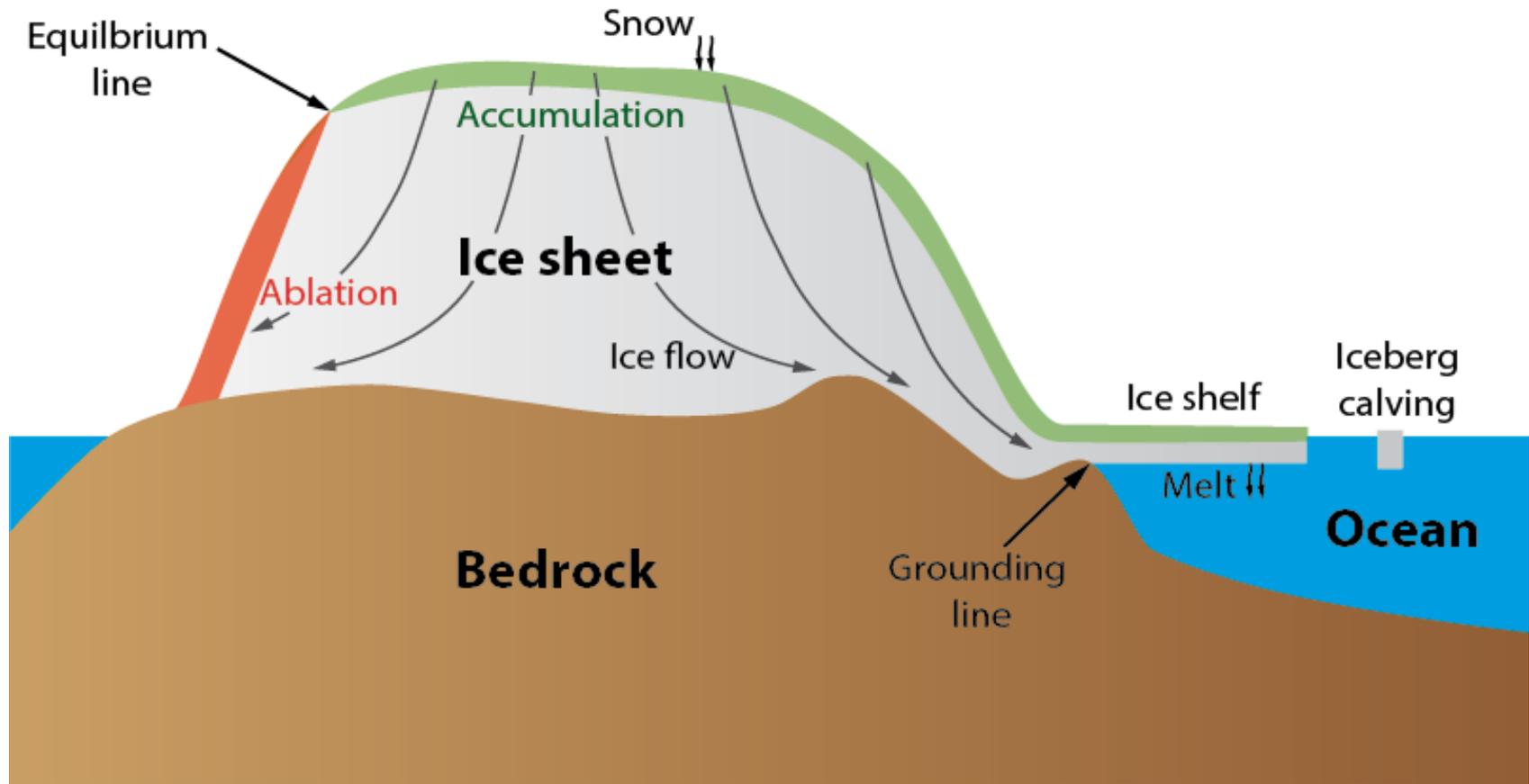
Local changes
in ice thickness

Term related to
transport

Local mass balance
(accumulation-melting)

where \vec{u}_m is the depth-averaged horizontal velocity field, H the thickness of the ice sheet and M_b is the mass balance, accounting for snow accumulation as well as basal and surface melting.

Components of a climate model: ice sheets



Some of processes included in an ice sheet model.

Many models compute the changes in atmospheric composition (aerosols, various chemical species) interactively.

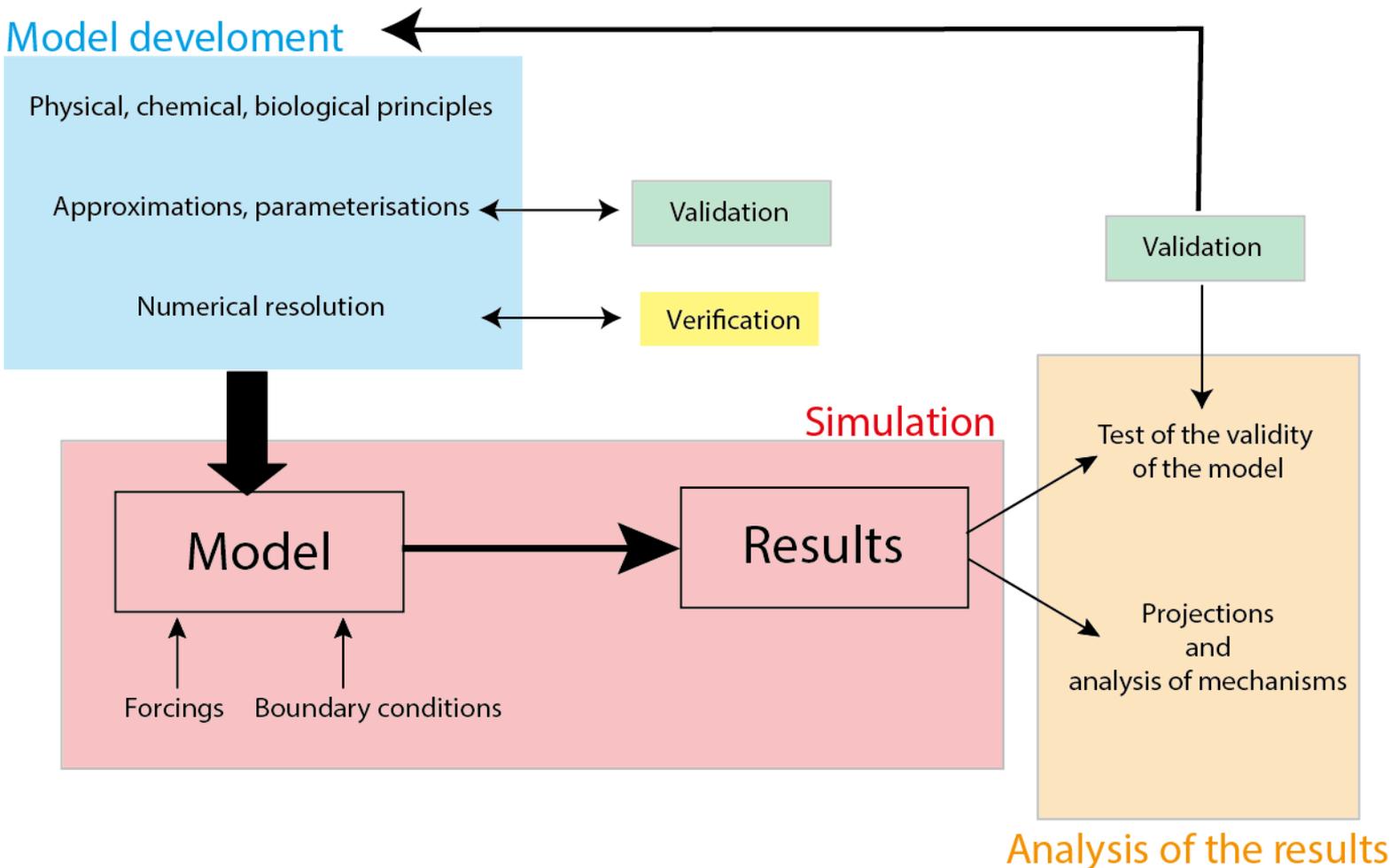
The interactions between the various components requires special care both on physical and technical aspects.

Model evaluation: verification, validation, calibration

Verification: the numerical model adequately solves the equations.

Validation: test if the model results are sufficiently close to reality

A validation is never complete.



Calibration: adjusting parameters to have a better agreement between model results and observations.

Calibration is justified as the value of some parameters is not precisely known.

Calibration should not be a way to mask model deficiencies.

Evaluating model performance

The goal of **performance metrics** is to provide an objective assessment of the performance of the model.

The simplest metrics are based on the difference between observations and model results:

$$RMSE = \sqrt{\frac{1}{n} \sum_{k=1}^n (T_{s,mod}^k - T_{s,obs}^k)^2}$$

where n is the number of grid points for which observations are available, $T_{s,mod}^k$ is the model surface air temperature at point k and $T_{s,obs}^k$ is the observed surface air temperature at the same point.

Standard tests and model intercomparison projects

The conditions of the standard simulations are often defined in the framework of Model Intercomparison Projects (MIPs).

Climate of the last 50-150 years

- + Mean state
- + Variability at all timescales
- + Climate changes over the last 150 years

Paleoclimate modelling

- + Last millenium and Holocene
- + Last Glacial Maximum
- + More distant past

Idealised test cases

- + 2 times CO₂ experiments
- + Water hosing experiments

Classical tests performed on climate models

Ability of models to reproduce the current climate

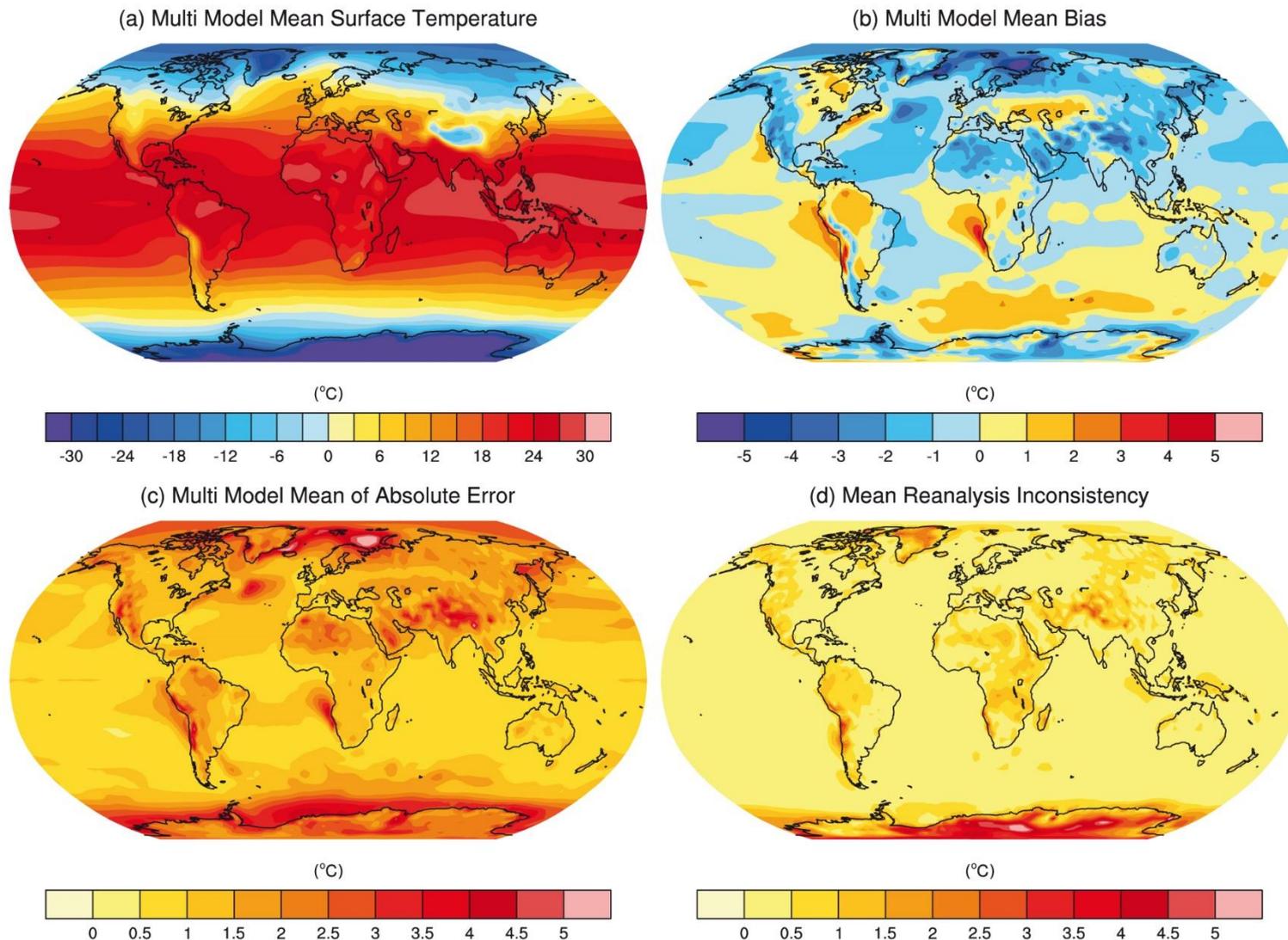
MIPs like CMIP5 (Coupled Model Intercomparison Project, phase 5) provide ensembles of simulation performed with different models.

The multi-model mean provides a good summary of the behavior of the ensemble.

The multi-model mean have usually smaller biases than individual members.

Ability of models to reproduce the current climate

Annual-mean surface temperature(°C)



Ability of models to reproduce the current climate

Annual-mean precipitation rate (mm day^{-1})

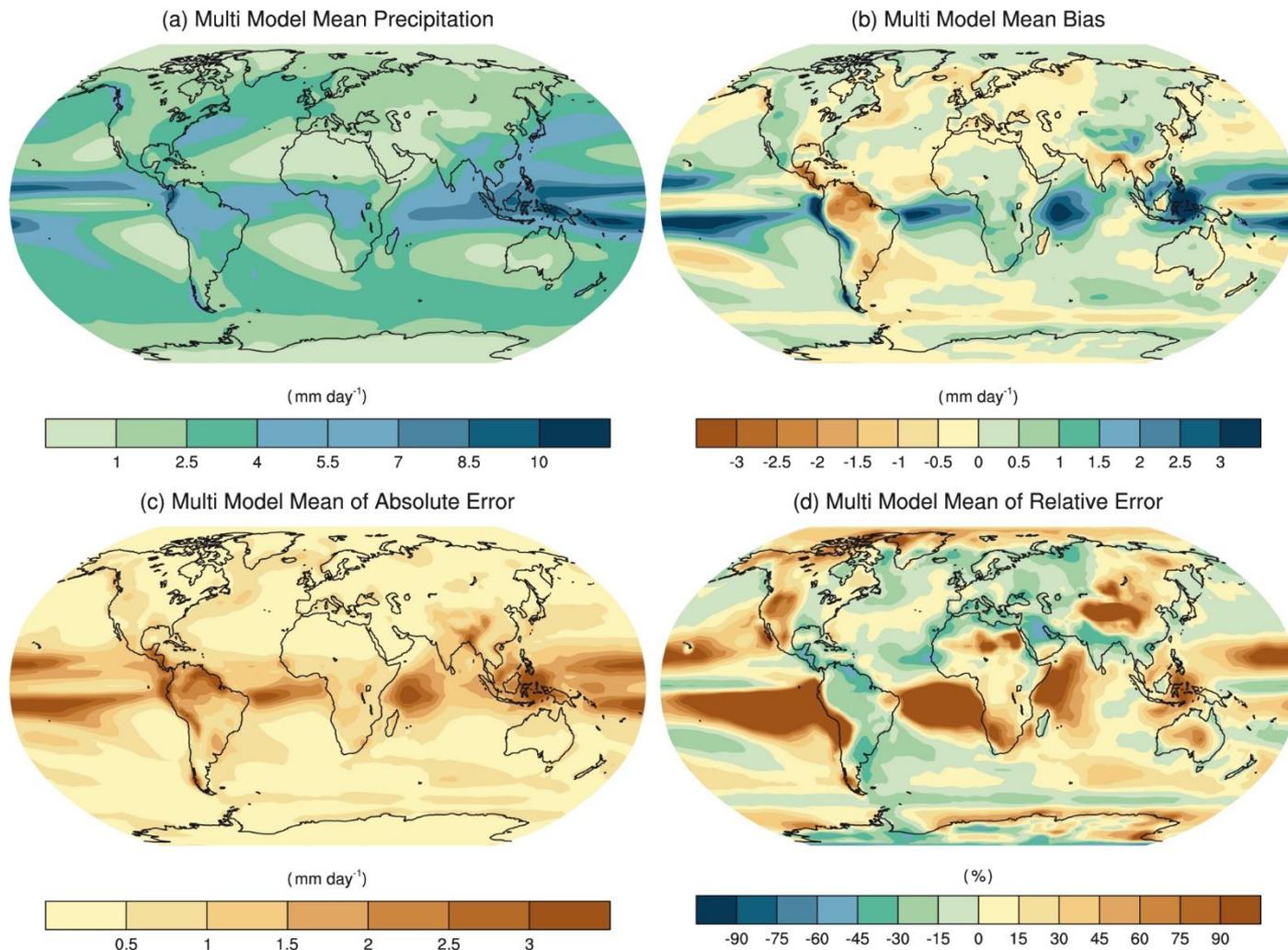
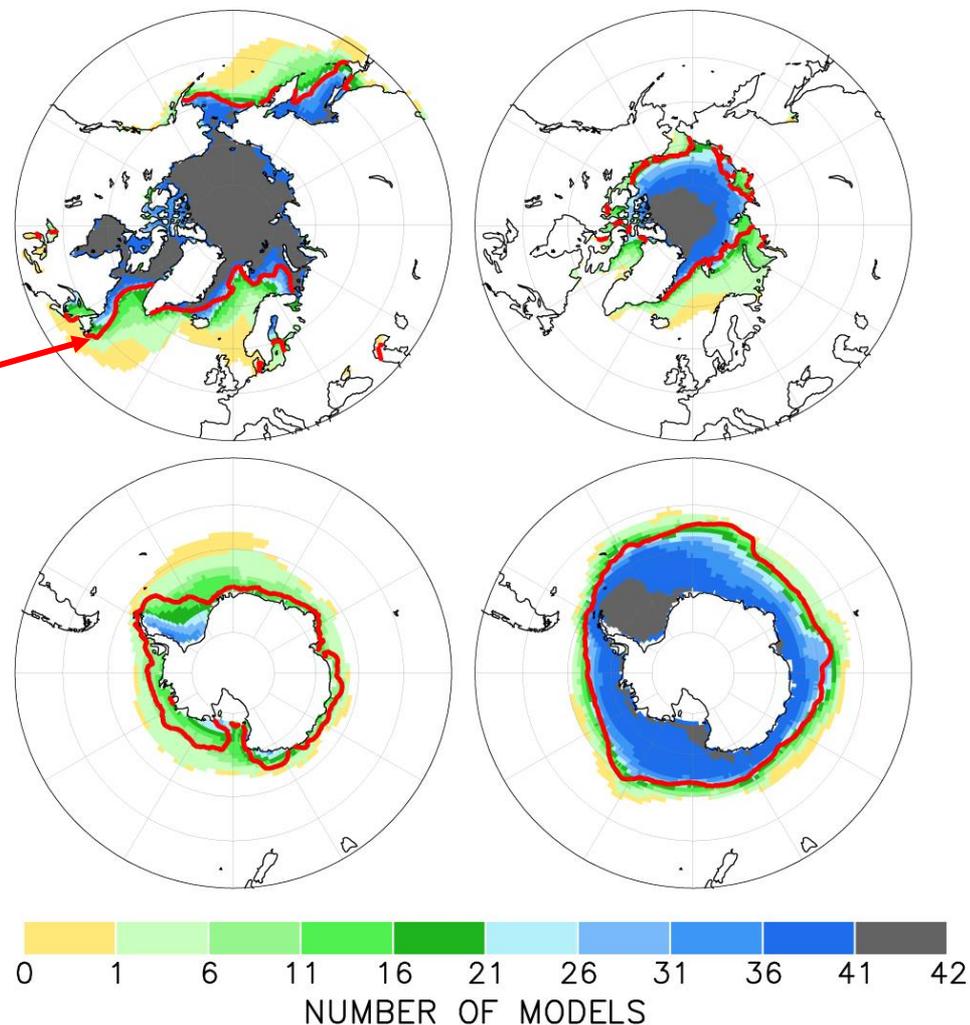


Figure from Flato et al. (2013).

Ability of models to reproduce the current climate

Sea ice distribution

Observed ice edge



The colors indicate the number of models that simulate at least 15% of the area covered by sea ice. The red line is the observed ice edge. Figure from Flato et al. (2013).

Correction of model biases

Model results have biases that need to be corrected for some applications.

Simplest assumption: the bias is constant

⇒ the correction is constant.

Mean of the simulated variable and of the observations over the reference period.

$$\begin{aligned} x_{corr}(t) &= x_{mod}(t) - (\bar{x}_{mod} - \bar{x}_{obs}) \\ &= \bar{x}_{obs} + \underbrace{(x_{mod}(t) - \bar{x}_{mod})}_{\text{anomaly}} \end{aligned}$$

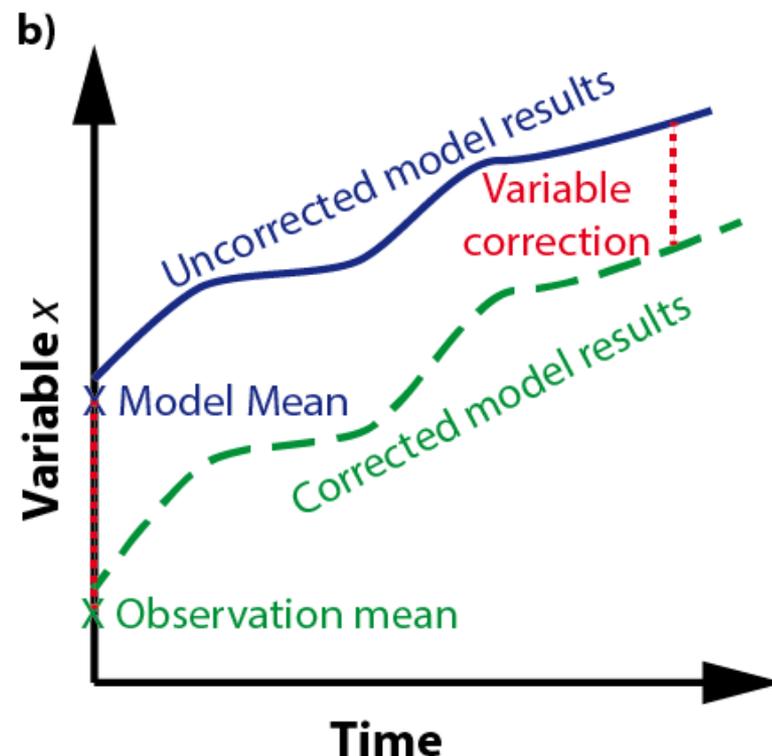
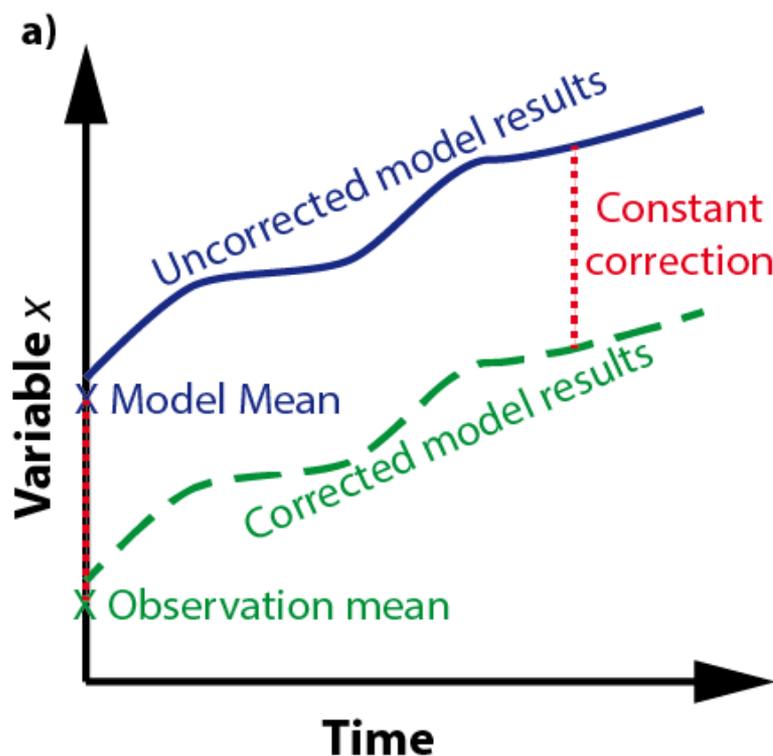
Corrected variable

Raw simulated variable

Correction of model biases

Correction based on previous simulations (in similar conditions) that have been compared with observations.

1/ Starting from a quasi-equilibrium of the model



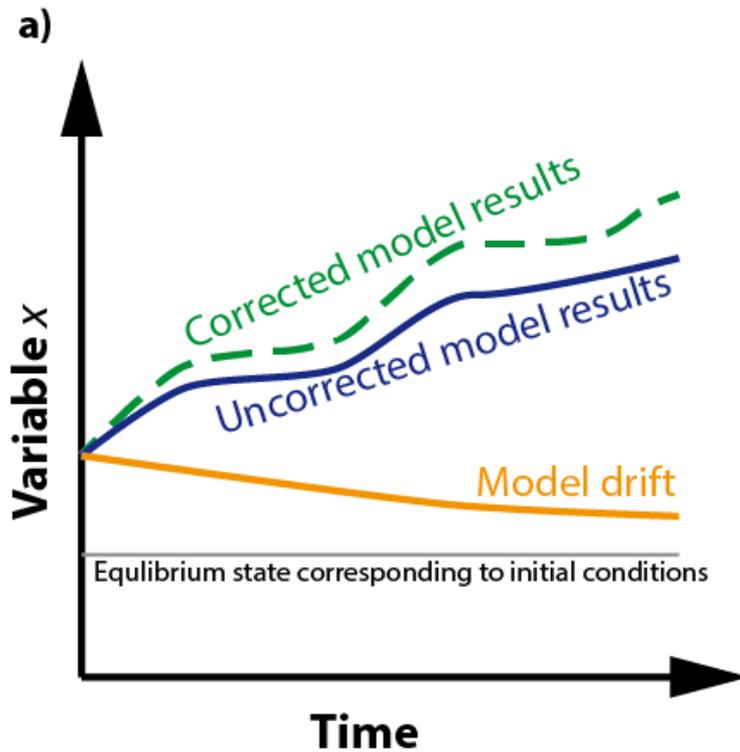
If a model has been shown to underestimate systematically the warming trend due to a particular forcing over past periods, this bias can be corrected by an amplification of the long term trend.

Correction of model biases

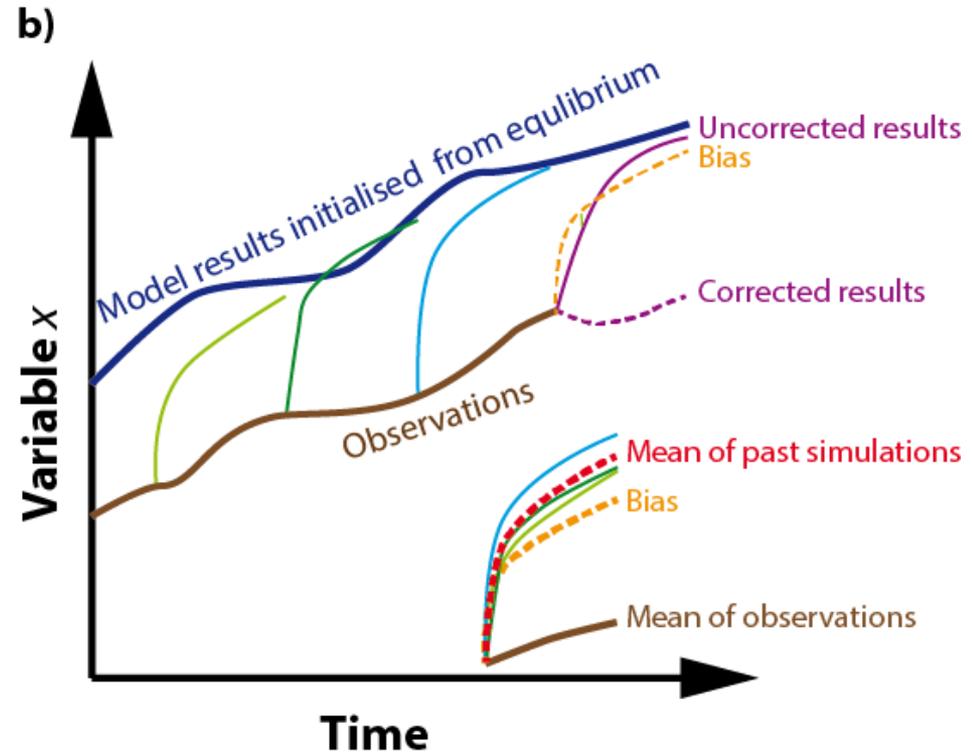
Non-stationary correction.

2/ Starting from a state far from the equilibrium of the model

Correction of model drift



Starting from a non-equilibrated initial state.

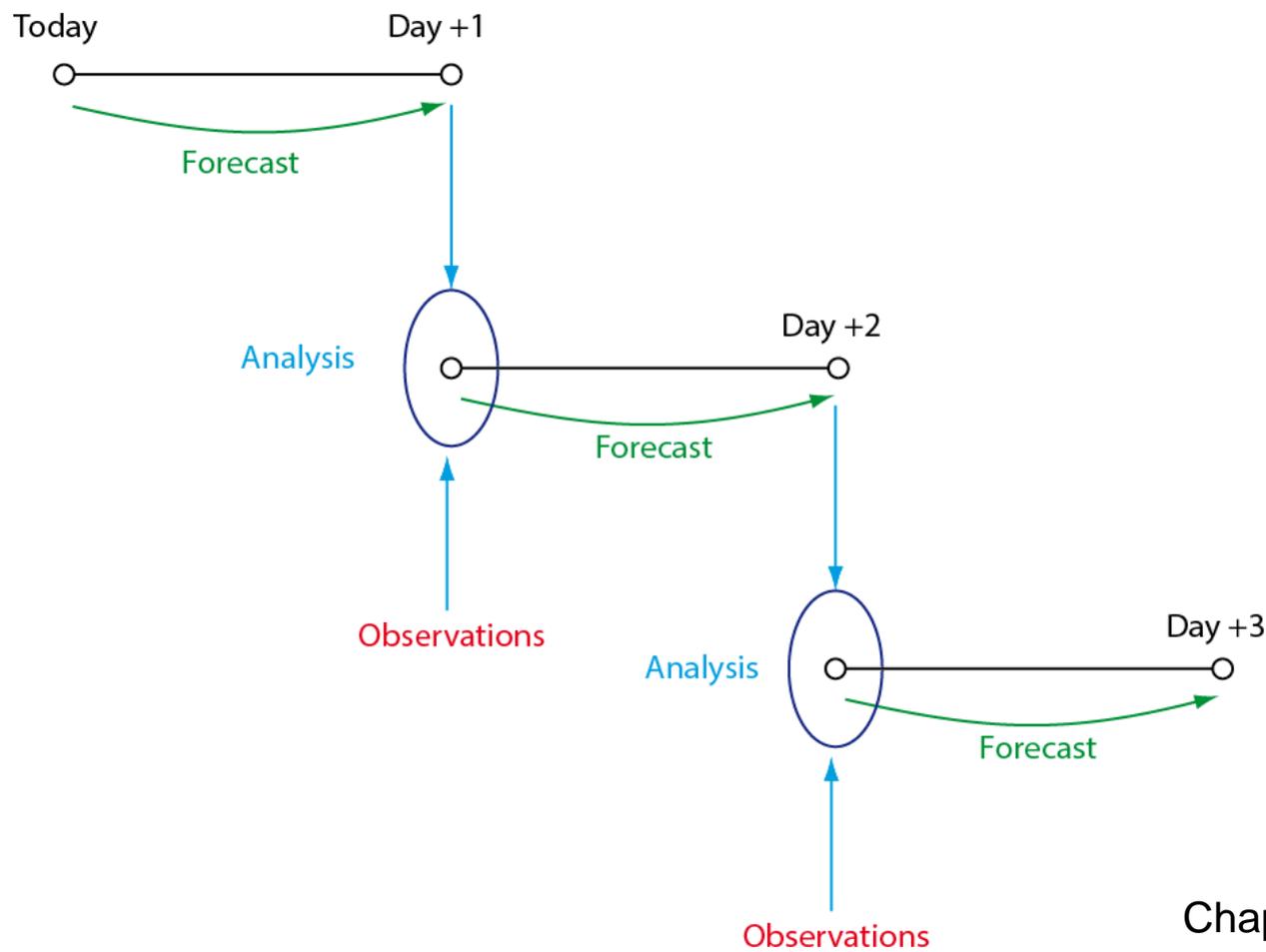


Starting directly from observations

Data assimilation

The main goal of data assimilation is to optimally combine model results and observations to estimate as accurately as possible the state of the system: *state or field estimation*.

Sequential data assimilation.



Sequential data assimilation

Simple **example**: two independent measurements of the same temperature $T_1(t)$ and $T_2(t)$ with error variances σ_1^2 and σ_2^2 .

Optimal estimate (analysis) T_a :

$$T_a(t) = w_1 T_1(t) + w_2 T_2(t)$$

with

$$w_1 = \frac{1/\sigma_1^2}{1/\sigma_1^2 + 1/\sigma_2^2}$$

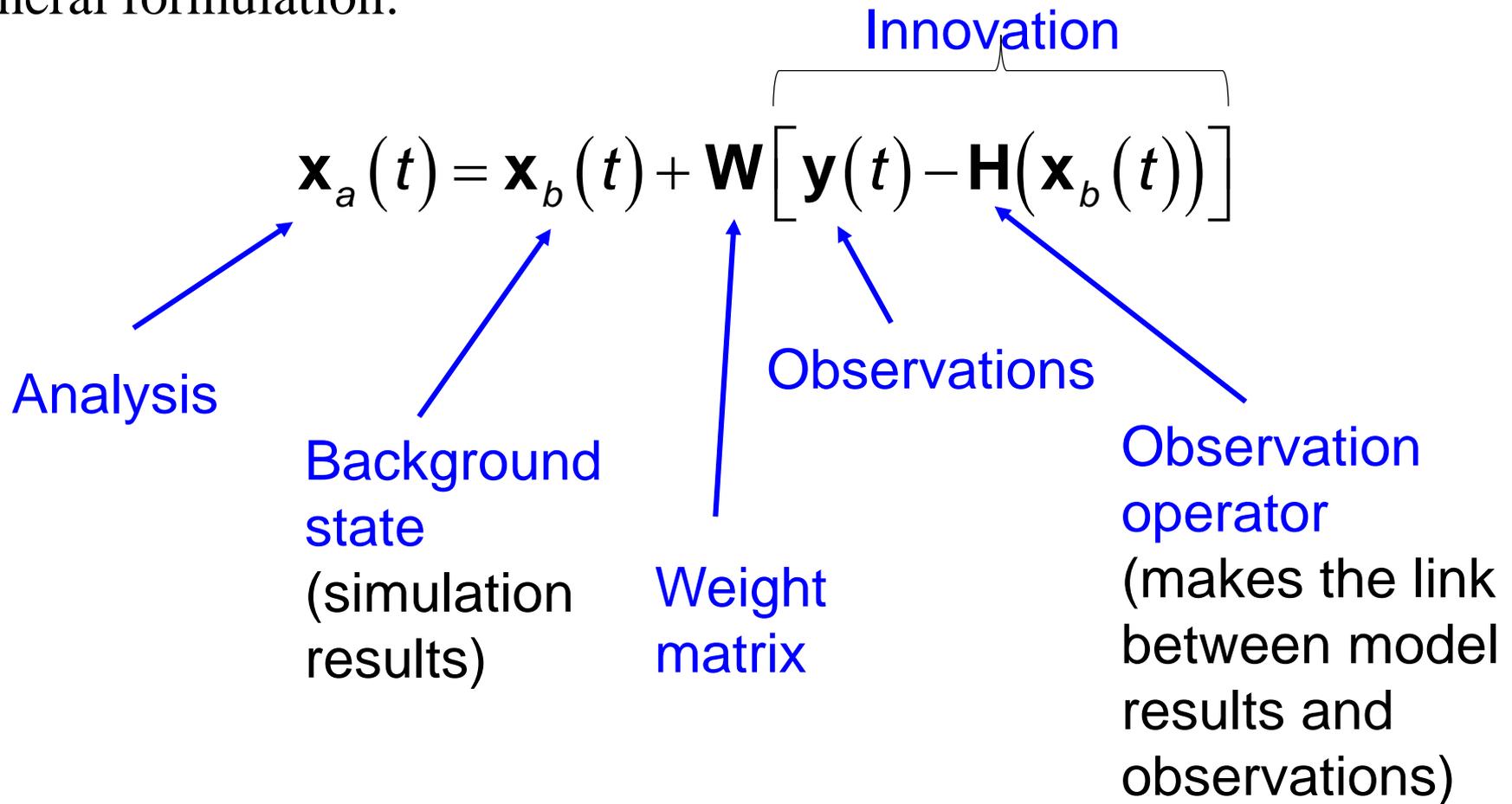
$$w_2 = \frac{1/\sigma_2^2}{1/\sigma_1^2 + 1/\sigma_2^2}$$

The more
“precise” time
series get the
largest weight.

Data assimilation

Sequential data assimilation

General formulation:



The weight matrix \mathbf{W} depends on the model and observation errors.